Technical Comments

Comment on Moretti's Method

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IN this method, all discontinuities of the flow variables and their first derivatives are taken as boundaries of separate domains of computation and, in Ref. 1, Moretti shows that under these circumstances the matrix form [Eq. (1)] of the equations governing the flow gives better results than the conservation form [Eq. (2)]:

matrix form

$$\begin{bmatrix} \ln p \\ u \\ S \end{bmatrix}_t + \begin{bmatrix} u & \gamma & 0 \\ p/\rho & u & 0 \\ 0 & 0 & u \end{bmatrix} \begin{bmatrix} \ln p \\ u \\ S \end{bmatrix}_x = 0 \tag{1}$$

conservation form

$$\begin{bmatrix} \rho \\ \rho u \\ e \end{bmatrix}_t + \begin{bmatrix} \rho u \\ \rho u^2 + p \\ u(e+p) \end{bmatrix}_x = 0 \tag{2}$$

where p = static pressure, u = velocity, ρ = density, S = entropy $= \ln p - \gamma \ln \rho$, e = energy $= \rho u^2/2 + p/(\gamma - 1)$, γ = ratio of specific heats.

The question is left open, if the set of variables adopted in Eq. (1), i.e., $\ln p$, u, S, is the best choice possible. It is not if computation time is considered, since in an electronic computer the calculation of the matrix element p/ρ [also needed to determine the time step $\Delta t = \text{const}\, \Delta x/(|u| + a)$, $a = (\gamma p/\rho)^{1/2}$] involves the use of an additional library function (the exponential) which is rather time-consuming compared with intrinsic functions.

To give an example we tried another set of variables (still in matrix form):

$$\begin{bmatrix} p \\ u \\ \rho \end{bmatrix}_t + \begin{bmatrix} u & \gamma p & 0 \\ 1/\rho & u & 0 \\ 0 & \rho & u \end{bmatrix} \begin{bmatrix} p \\ u \\ \rho \end{bmatrix}_x = 0 \tag{3}$$

which allows to compute all quantities by simple intrinsic arithmetic.

For comparability, we used Moretti's test case from Refs. 1 and 2, the piston-driven flow (with zero initial acceleration of the piston; $\Delta x = 0.025$). The difference scheme was MacCormack's (second-order accuracy; No. 7 in Ref. 1, p. 42). By introducing additional arrays for storing p/ρ and the auxiliary values in MacCormack's scheme we completely avoided multiple computations, but nevertheless the net calculation time per nodal point on a UNIVAC 1108 was 371 μsec for the set of variables (1) and 278 μsec for the set (3), which is a ratio of 1.333 to 1.† Taking into account additionally the organizing loops of the main program this ratio reduces slightly, in a typical case to 1.30 to 1.

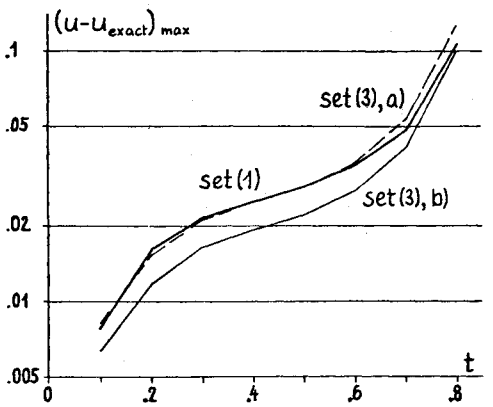

Fig. 1 Comparison of errors.

This advantage in speed can be used in two ways: a) We can accept it directly as a saving of 30% in computation cost at a very slight sacrifice in accuracy—see curve a in Fig. 1, which gives the maximum velocity error as a function of physical time (and is a close relative of Fig. 10 in Ref. 1); or b) we can readjust the computation parameters Δx and/or $(\Delta t/\Delta x)(|u| + a)$ for a computation time equal to set (1) and accordingly better accuracy. Curve b shows the recommendable case of unchanged Δx and $(\Delta t/\Delta x)(|u| + a)$ decreased 30%.

References

[1] Moretti, G., "The Choice of a Time-Dependent Technique in Gas Dynamics," PIBAL Rept. 69-26, July 1969, Polytechnic Inst. of Brooklyn, Brooklyn, N.Y.

[2] Moretti, G., "A Critical Analysis of Numerical Techniques: The Piston-Driven Inviscid Flow," PIBAL Rept. 69-25, July 1969, Polytechnic Inst. of Brooklyn, Brooklyn, N.Y.

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† Measuring CPU-time in machines using cycle-stealing is not easy to manage; the machine must be empty save the one program under test. An easier way consists in evaluating an assembler listing. Differences between measured and evaluated times were found negligible.